Area of an Ellipse Proof by Matthew Wraith

a = Major axis of the ellipse
b = Minor axis of the ellipse
x = x value of the ellipse
y = y value of the ellipse
A = Area of the ellipse

Trig substitution:

$$x = a\,\sin\theta$$
$$dx = a\,\cos\theta\,d\theta$$
$$New\,bounds:\ 0\,to\,\frac{\pi}{2}$$

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \tag{1}$$
$$\frac{b^2x^2}{a^2}+y^2=b^2 \tag{2}$$
$$y^2=b^2-\frac{b^2x^2}{a^2} \tag{3}$$
$$y=\sqrt{b^2-\frac{b^2x^2}{a^2}} \tag{4}$$
$$y=\frac{b}{a}\sqrt{a^2-x^2} \tag{5}$$

$$\begin{aligned}
A &= 4\frac{b}{a}\int_0^a \sqrt{a^2-x^2}dx \\
&= 4\frac{b}{a}\int_0^{\frac{\pi}{2}} \sqrt{a^2-a^2\sin^2\theta}\,a\cos\theta\,d\theta \\
&= 4\frac{b}{a}\int_0^{\frac{\pi}{2}} \sqrt{a^2\cos^2\theta}\,a\cos\theta\,d\theta \\
&= 4ab\int_0^{\frac{\pi}{2}} \cos^2\theta\,d\theta \\
&= 4ab\left(\frac{\sin\theta\cos\theta+\theta}{2}\right)\Big|_0^{\frac{\pi}{2}} \\
&= 2ab\left(\frac{\pi}{2}\right)-2ab(0) \\
&= \pi ab
\end{aligned} \tag{6}$$